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A New Method of Construction of Second Order Slope Rotatable Designs using a Pair of Incomplete Block Designs

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SUMMARY

A new method of construction of three and five level second order slope rotatable designs (SOSRDs) using two suitably chosen balanced incomplete block desings (BIBDs) is suggested.

Key words : Response surface designs; Slope rotatability; Second order slope rotatable designs.

Introduction

Hader and Park [1] introduced slope rotatable central composite designs. Victorbabu and Narasimham [2], [3], [4] studied in detail the conditions to be satisfied by a general SOSRD and also constructed SOSRDs using BIB designs. For definitions and notations refer to Hader and Park [1] and Victorbabu and Narasimham [2]. It is clear from slope rotatability condition (in page 2471 of Victorbabu and Narasimham [2]) that the solutions for design levels (like a, b, etc.,) depend on c and n_0. In contrast in rotatable design the solutions for design levels a, b, etc., in second order rotatable design do not depend on the number of central points n_0. This interdependence of the parameters n_0, c and design levels in SOSRD is utilised to evolve a new method for construction.

In SOSRD usually the number of central points n_0 is pre-fixed and design levels are chosen to satisfy the condition of SOSRD. The parameter c is then evaluated. Alternatively we may pre-fix c and choose n_0 and the design levels suitably such that the design points satisfy the conditions of SOSRD. If such designs exist with integral values for n_0, then this approach leads to a new method of construction of SOSRD.

2. *Three level SOSRDs using a pair of BIBDs*

A new method of construction of three levels SOSRDs using two suitably chosen BIB designs without any additional set of points, can be obtained as follows.

Let $D_1 = (v, b_1, r_1, k_1, \lambda_1)$, and $D_2 = (v, b_2, r_2, k_2, \lambda_2)$ are two BIB designs ($2^{t(k_1)}$ and $2^{t(k_2)}$ denote Resolution V fractional replicates of 2^{k_1} and 2^{k_2} factorials with levels ± 1) in V treatments with $r_1 \lesseqgtr c\lambda_1$ and $r_2 \gtreqless c\lambda_2$ (c pre-fixed) respectively. Let $[a - (v, b_1, r_1, k_1, \lambda_1)]\, 2^{t(k_1)}$ denote the $b_1\, 2^{t(k_1)}$ design points, $[a - (v, b_2, r_2, k_2, \lambda_2)]\, 2^{t(k_2)}$ denote the $b_2\, 2^{t(k_2)}$ design points generated from the BIB designs by multiplication (c.f. Victorbabu and Narasimham [2], [3]).

Three level SOSRDs using BIB designs can be constructed as follows. Repeat the set of $b_2\, 2^{t(k_2)}$ design points generated from BIBD–D_2 y-times. Then with the above design points along with n_0 central points we can construct a three level SOSRD as given in the following Theorem 2.1.

Theorem 2.1 : The design points,

$[a - (v, b_1, r_1, k_1, \lambda_1)]\, 2^{t(k_1)} \cup y[a - (v, b_2, r_2, k_2, \lambda_2)]\, 2^{t(k_2)} \cup (n_0)$ give a three level v-dimensional SOSRD in

$N = b_1\, 2^{t(k_1)} + y\, b_2\, 2^{t(k_2)} + n_0$ design points, if $(r_1 - c\lambda_1)(r_2 - c\lambda_2) \le 0$,

$$y = \frac{(c\lambda_1 - r_1) 2^{t(k_1) - t(k_2)}}{(r_2 - c\lambda_2)}, \tag{2.1}$$

$$n_0 = \frac{-[v(c-5)+4]c^2[\lambda_1\, 2^{t(k_1)} + y\lambda_2\, 2^{t(k_2)}]}{[v(5-c)-(c-3)^2]} - b_1\, 2^{t(k_1)} - y\, b_2\, 2^{t(k_2)} \tag{2.2}$$

and n_0 turns out to be an integer.

Proof : For the design points generated from BIBD–D_1 and y-repetetions of points from BIBD–D_2, conditions of SOSRD are true as follows.

2. (i) $\sum x_{iu}^2 = r_1 2^{t(k_1)} a^2 + y r_2 2^{t(k_2)} a^2 = N\lambda_2 = \text{constant}$

(ii) $\sum x_{iu}^4 = r_1 2^{t(k_1)} a^4 + y r_2 2^{t(k_2)} a^4 = cN\lambda_4 = \text{constant}$

3. $\sum x_{iu}^2 x_{ju}^2 = \lambda_1 2^{t(k_1)} a^4 + y\lambda_2 2^{t(k_2)} a^4 = N\lambda_4 = \text{constant}$ (2.3)

From 2(ii) and (3) of (2.3), we have

$$r_1 2^{t(k_1)} a^4 + y\, r_2 2^{t(k_2)} a^4 = c\left[\lambda_1 2^{t(k_1)} a^4 + y\,\lambda_2 2^{t(k_2)} a^4\right]$$

which leads to y given in (2.1). Equation (2.1) has a real solution only if $(r_1 - c\lambda_1)(r_2 - c\lambda_2) \leq 0$. From the slope rotatability condition, we have

$$\frac{\lambda_4}{\lambda_2^2} = \frac{-[v(c-5)+4]}{[v(5-c)-(c-3)^2]} \tag{2.4}$$

which leads to n_0 given in (2.2) using (2.3). The value of a can be obtained from 2(i) of (2.3).

Corollary : Taking $D_1 = (v = v, b_1 = b, r_1 = r, k_1 = k, \lambda_1 = \lambda)$ and $D_2 = (v = v, b_2 = v, r_2' = 1, k_2 = 1, \lambda_2 = 0)$ in the above theorem (2.1), we get Victorbabu and Narasimham [4] method of construction of three level SOSRD using BIB design in $N = b\,2^{t(k)} + 2yv + n_0$ design points.

Example : Consider the design points,

$[a-(7,7,3,3,1)]\,2^3 \cup y[a-(7,21,6,2,1)]\,2^2 \cup (n_0)$ will give a three level 7-factor SOSRD in N = 600 design points with c = 5, y = 8 and n_0 = 32.

3. *Five level SOSRD using a pair of BIB designs*

A new method of construction of five level SOSRD using two suitably chosen BIB designs without any additional set of points, can be obtained as follows.

Let $D_1 = (v, b_1, r_1, k_1, \lambda_1)$, and $D_2 = (v, b_2, r_2, k_2, \lambda_2)$ are two BIB designs in v treatments with $r_1 \lesseqgtr c\lambda_1$ and $r_2 \gtreqless c\lambda_2$ (c pre-fixed). The method of construction is given in the following theorem (3.1).

Theorem (3.1) : The design points,

$[1-(v, b_1, r_1, k_1, \lambda_1)]\,2^{t(k_1)} \cup [a-(v, b_2, r_2, k_2, \lambda_2)]\,2^{t(k_2)} \cup (n_0)$ give a five level v-dimensional SOSRD in

$N = b_1 2^{t(k_1)} + b_2 2^{t(k_2)} + n_0$ design points, if $(r_1 - c\lambda_1)(r_2 - c\lambda_2) \leq 0$,

$$a^4 = \frac{(c\lambda_1 - r_1)2^{t(k_1) - t(k_2)}}{(r_2 - c\lambda_2)}, \tag{3.1}$$

$$n_0 = \frac{-[v(c-5)+4]\left[r_1\, 2^{t(k_1)} + r_2\, 2^{t(k_2)}a^2\right]^2}{[v(5-c)-(c-3)^2]\left[\lambda_1\, 2^{t(k_1)} + \lambda_2\, 2^{t(k_2)}a^4\right]} - b_1\, 2^{t(k_1)} - b_2\, 2^{t(k_2)} \tag{3.2}$$

and n_0 turns out to be an integer.

Proof : For the design points generated from BIBD–D_1 and BIBD–D_2, conditions of SOSRD are true as follows.

2. (i) $\sum x_{iu}^2 = r_1 2^{t(k_1)} + r_2 2^{t(k_2)} a^2 = N\,\lambda_2 = \text{constant}$

   (ii) $\sum x_{iu}^4 = r_1 2^{t(k_1)} + r_2 2^{t(k_2)} a^4 = cN\,\lambda_4 = \text{constant}$

3. $\sum x_{iu}^2 x_{ju}^2 = \lambda_1\, 2^{t(k_1)} + \lambda_2\, 2^{t(k_2)} a^4 = N\,\lambda_4 = \text{constant}$ (3.3)

From 2(ii) and (3) of (2.3), we have

$$r_1 2^{t(k_1)} + r_2 2^{t(k_2)} a^4 = c\left[\lambda_1\, 2^{t(k_1)} + \lambda_2\, 2^{t(k_2)}\, a^4\right]$$

which leads to a^4 given in (3.1). Solving the slope rotatability condition, we get n_0 given in (3.2).

Example : The design points,

$[1-(7, 7, 3, 3, 1)]\, 2^3 \cup [a-(7, 21, 6, 2, 1)]\, 2^2 \cup (n_0)$ will give a five level 7-dimensional SOSRD in N = 216 with c = 5. Here (3.1) leads to a = 1.4142 and (3.2) leads $n_0 = 76$.

It can be easily seen that by taking higher value of c (≥ 5), we get designs with lesser number of design points.

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Predicting the monthly rainfall around Guwahati using a Seasonal ARIMA model

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SUMMARY

Monthly rainfall at Guwahati is modelled using a Seasonal ARIMA series. The model parameters are estimated using Marquardt algorithm for non-linear optimization. The various stages of model building are presented in a simple algorithmic form. The model is used to predict rainfall for the month ahead and monthwise rainfall for the year ahead.

Key Words : Seasonal ARIMA model, acf, pacf, periodogram, white noise, Marquardt Algorithm, Kolmogorov-Smirnov test.

Introduction

Rainfall plays the most important role in the agricultural economy of Assam. Almost all the total available supply of water for growing crops in the state is met by rainfall. Though heavy monsoon rainfall occurs every year in Assam, it varies from season to season, year to year and place to place. For example, the mean annual rainfall during the years 1951-1976 was 181.24 cm in Kamrup district with a standard deviation of 53.48 cm, whereas the same was 212.99 cm in Sibsagar district with a standard deviation of 29. 36 cm. There have been statistical studies [2], [3], [4] on the rainfall in India and its geographical regions. Parthasarathy and Mooley [2] have constructed a summer monsoon (June to September) rainfall series for India as a whole for the period 1866–1970. On the basis of application of Eisenhart's run test and Mann-Kendall rank statistic test, they have found that the series neither shows any significant trend nor any significant oscillations. Power spectrum analysis of the series has been reported to indicate a weak Quasi Biennial Oscillation (Period 2.3 to 2.8 years).

While most of the studies deal with the annual rainfall, we attempt modelling the monthly rainfall pattern of Guwahati using a Univariate Box - Jenkins (UBJ) model and predict the rainfall ahead of one month and one year. This study can be useful in predicting the occurrence of flood, planning of irrigation schemes and crop plantation in the agricultural belt around